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Research Article

Gradient-Based MPC Controller for One DOF Copter Control

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ABSTRACT

This study tries to contrast the performance of Gradient Based Model Predictive Control (MPC) with the classical PID controllers in the control of a 1 DOF copter system. While PID controllers are easy to set up and be applied in a vast number of occasions, they may become insufficient when nonlinear systems or operating constraints are considered. To that end, results show that MPC, whose optimization formulation predicts the system dynamics, has better performance measures. Simulation results show that the gradient-based MPC performs well in dynamic and steady-state performance compared to PID controllers tuned with two different coefficients. In particular, the MPC is characterized by high accuracy and minimum long-term error, no overshoot and fast settling time. In noisy conditions, MPC is able to produce a stable control signal due to its predictive capability, whereas PID controllers are more sensitive to such conditions and produce a chattering signal. It is concluded that the Gradient Based MPC is a good choice for applications that require more precise control and robustness against noise, while PID controllers are preferable due to their simplicity and low computational cost.

1. Introduction

Control systems are essential to the performance and stability of dynamic systems, especially in robotics and aerospace applications where accuracy, dependability, and efficiency are crucial. The capacity of unmanned aerial vehicles (UAVs) or copter systems to remain stable in the face of disruptions and execute moves with the least amount of error is determined by the control mechanism. The most popular control techniques for these kinds of applications are Model Predictive Control (MPC) and Proportional-Integral-Derivative (PID) controllers.

Proportional-Integral-Derivative (PID) controllers generate control signals by comparing the current state of a system with a desired reference value and adding proportional (P), integral (I) and derivative (D) components to this error [1]. Nonlinear systems may require more advanced methods such as MPC. MPC is a method that predicts future behavior using a system model and determines control signals by solving an optimization problem in each control loop. This approach offers a control performance that can better handle system dynamics and effectively manage constraints [2].

Because of their simplicity, convenience of use, and adequate performance in a variety of linear and time-invariant systems, PID controllers have long been the industry standard. However, nonlinear systems or processes with restrictions may be difficult for PID controllers to handle; in these cases, improved control algorithms such as MPC have proven to perform better. MPC is a model-based method that can better manage system restrictions and nonlinearity than PID since it forecasts future system behavior and resolves an optimization issue at each control interval [3]. It has been demonstrated that MPC performs better than PID in complicated systems, including multi-degree-of-freedom (DOF) copters, in terms of response time, overshoot, and energy efficiency [4].

PID and MPC controllers have been compared in numerous research for a range of applications. For instance, a comparison of MPC and optimized PID controllers for first through fifth-order systems was carried out by Salem et al. (2015). Their findings repeatedly demonstrated that MPC was a better option for systems with complicated dynamics

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than PID in terms of rise time, settling time, and overshoot [5]. Similarly, Sharma and Pfeiffer (2017) showed that MPC, as opposed to standard PID, offered smoother control inputs and greater overall system stabilization in the control of a 2-DOF helicopter, particularly when managing cross-coupling effects n multivariable systems [6].

Benotsmane and Kovács (2023) used both traditional PID and MPC controllers to study how much energy industrial robots use in industrial settings. Their research revealed that PID retained greater accuracy in trajectory tracking, whereas MPC was more energy-efficient, consuming less across complex motion routes [7]. These findings imply that PID may still provide benefits in terms of ease of use and accuracy for particular jobs, even while MPC can maximize performance and efficiency.

Control systems are also commonly faced with many types of noise and disturbances in real-world applications, which can seriously impair system performance. Instability and a decline in the overall accuracy of the control system might result from noise, which can be caused by hardware limits, external influences, or inaccurate sensors [8]. For example, in high-precision dynamic systems, external noise can result in oscillations, overshoot, or prolonged settling times, thereby reducing the system's robustness and reliability. It is therefore essential to assess how control strategies such as MPC, PID, and feedforward PID respond to noise in order to gain insight into their practical applicability in real-world scenarios. As demonstrated by studies conducted by Efheij et al. (2019), while feedforward PID can enhance system responsiveness in the presence of disturbances, MPC frequently exhibits superior performance in managing dynamic noise conditions due to its predictive nature and capacity to optimize control actions over time.

Because of their simplicity and capacity to illustrate important control concepts, 1 DOF copter models provide a fundamental platform for testing and creating control algorithms. Before advancing to more intricate multi-DOF systems, these systems are frequently used to test basic control strategies like PID control. For example, Karam et al. (2022) used a PID controller to develop and implement a 1 DOF drone, stabilizing the system with 97% accuracy. An excellent testbed for improving control strategies that can subsequently be used with multi-rotor copters is provided by this platform [10].

A study on the mathematical modeling and control of a one-degree-of-freedom quadcopter system was given by Jafar et al. (2016). The study investigated the link between thrust and rotor RPM using closed-loop PID control, confirming the control method through real-time testing [11]. In this regard, the 1 DOF copter is very helpful since it offers a simplified setting for analyzing dynamic control and stabilization performance.

In order to gain insight into flight dynamics and validate control strategies, Ahmad (2007) investigated the aerodynamic modeling and real-time control of a 1 DOF tailplane system. According to the study, these technologies are useful for creating precise plant models for control system design and real-time use in unmanned aerial vehicles [12].

This paper presents a performance evaluation of an up-to-date method on the control of the One dof copter system, which is frequently used in the literature as an example of control action of unmanned aerial vehicles. The methods are tested with error-based performance metrics and time domain parameters under both noiseless and noisy conditions. Thus, it is aimed to compare the performance of Gradient-based MPC and classical PID and to reveal the advantages of each controller

2. Materials and Methods

2.1. One DOF Copter Model

In order to conduct a dynamic analysis of a helicopter system with one degree of freedom (1-DoF), the scheme depicted in Figure 1 is utilized. The system is represented by a rod attached to a fulcrum with a direct-current (DC) motor positioned at its end.

The rod is equipped with a motor of mass mh, situated at a distance lh from the fulcrum. The vertical force (Fh) generated by the motor ensures the dynamic equilibrium of the system. This force is contingent upon the motor torque and electrical dynamics. The acceleration of gravity (g) initiates the downward motion of the mass.

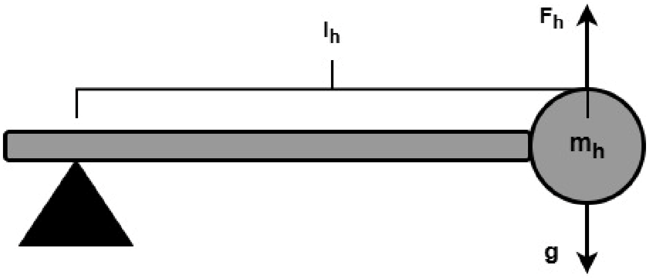


Figure 1: Model of the 1-DOF copter

The dynamic behavior of the system is described by the rotational motion of the rod and the ability of the motor to generate torque. The moment of inertia (J) of this system is expressed by the mass m_h and the distance l_h as:

$$J = m_h {l_h}^2 \tag{1}$$

The dynamic equation is expressed in the following form, derived by the Newton-Euler method:

$$J\ddot{\theta} + b\dot{\theta} = F_h l_h - m_h g l_h \tag{2}$$

In this equation:

$\ddot{\theta}$: *Angular acceleration of the rod*
$\dot{\theta}$: *Angular velocity of the rod*
b: *Viscous friction coefficent*

In the system model, the torque contribution of the part situated to the left of the pivot point in the stabilizing direction, along with the aerodynamic friction effects, have been disregarded. Given that the impact of gravity will fluctuate in accordance with the angle of the rod in relation to the ground, and given that it would introduce a trigonometric element to the equation, rendering the equation non-linear, this effect has not been incorporated into the system model.

The objective of this system is to stabilize the bar at a given reference angle by controlling the motor torque (T_h) and, consequently, F_h. The dynamics of the motor are represented by the following equations:

$$F_h = K_t I_m \tag{3}$$

$$V_m = R I_m + K_e \dot{\theta} \tag{4}$$

where;

K_t is the torque constant of the motor, V_m is the motor voltage, I_m is the current applied to the motor, R is the electrical resistance of the motor and K_e is the electromotive force constant.

The rotational torque generated by the motor at the pivot point is calculated according to the following formula:

$$T_h = F_h l_h = K_t I_m l_h \tag{5}$$

If we extract the expression Im from Equation 4 and substitute it into Equation 2:

$$J\ddot{\theta} + b\dot{\theta} = K_t l_h \frac{V_m - K_e\dot{\theta}}{R} \tag{6}$$

Laplace transform is taken for Equation 6 and if the adjustment is made:

$$Js^2\theta(s) + \left(b + \frac{l_h K_t K_e}{R}\right) s\theta(s) = \frac{l_h K_t}{R} V_m(s) \tag{7}$$

The transfer function is obtained as follows:

$$\frac{\theta(s)}{V_m(s)} = \frac{\frac{l_h K_t}{R}}{Js^2 + \left(b + \frac{l_h K_t K_e}{R}\right) s} \tag{8}$$

The state space model of the system with angle and angular velocity as state variables is obtained as follows:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{\left(b + \frac{l_h K_t K_e}{R}\right)}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\frac{l_h K_t}{R}}{J} \end{bmatrix} V_m \tag{9}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The parameters used for the system are presented in Table 1.

Table 1: System Parameters

System Parameters	
m_h	0.25 kg
l_h	0.2 m
K_e	0.006 V/(rad/s)
K_t	0.006 Nm/A
R	16.9 ohm
b	0.01 Ns/rad

2.2. Control Methods

Control of the system is realized with the Gradient-Based MPC method. In order to evaluate the performance of this method as an alternative to the classical PID controller, the PID controller method was tested with different coefficients.

Response Time and Transient Behavior parameters were taken into consideration while determining the coefficients of the PID controller. No limitation is applied for the control signal generated by the controller. The response time of the system is calculated according to the open loop bandwidth and the placement of the poles in the complex coordinate system. The transient behavior value is a measure of how aggressive the system will behave when reaching the reference value. This value is calculated using the open-loop damping ratio and natural frequency values.

In order to compare the two control methods, the PID coefficients were adjusted to show aggressive (faster response) and optimal behavior. The preferred coefficients are presented in Table 2. The step response obtained with PID coefficients is presented in Figure 2.

Table 2: Coeffecients of two PID Controllers

PID Coeffecients		
PID-1	Response Time: 0.1123 s Transient Behavior: 0.45	K_P: 792.1391 K_I: 1079.9804 K_D: 145.2536
PID-2	Response Time: 0.1123 s Transient Behavior: 0.9	K_P: 148.4306 K_I: 37.08 K_D: 148.5412

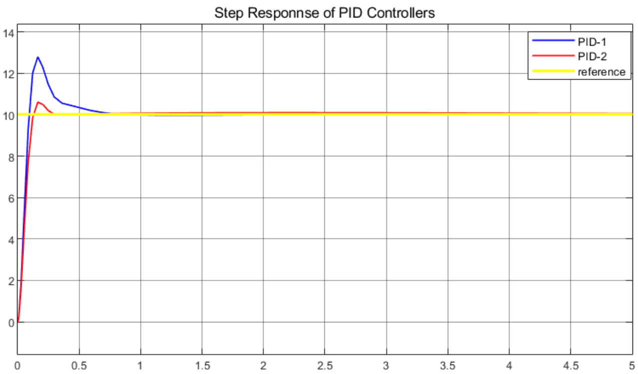


Figure 2: Step Responses of two PID Controllers

Gradient-Based MPC is a method that computes optimal control signals by estimating the future state of the system in control problems. This approach is characterized by its flexibility and performance, especially in nonlinear or time-varying systems. The basic parameters of this control method are presented as follows:

Prediction Horizon: As a result of the experiments performed on the state space model, the prediction horizon was chosen as 15. This value means that the control algorithm will optimize the future states of the system by looking 15 steps ahead.

Optimization Method: The calculation of the control signal is performed with Matlab's fmincon function. This function uses a gradient-based interior point algorithm to minimize the cost function. This method iteratively minimizes the objective function by continuing the search for solutions in the interior regions of the constraints rather than at the edges [13]. To ensure fast convergence, especially for large and complex problems, barrier functions are used to prevent violations of the constraints and move towards the optimal solution. The control signal constraints added to the optimization problem represent the physical boundaries of the system. While determining the constraints of the control signal, firstly, the control signal obtained by operating the PID controllers without applying saturation was analyzed and the maximum value was recorded. This value was set as the constraint value of the optimization and the method was made to work with equal constraints.

Cost Function: It is designed to ensure that the system states are close to the reference states and that the control signal is not unnecessarily large. For this purpose, the following function is used, which consists of two basic components and coefficients related to the square of the error and the control signal.

$$\boldsymbol{CF} = \sum_{k=1}^{N} \left[(x_k - x_{kref,k})^T Q (x_k - x_{ref,k}) + u_k^T R u_k \right] \quad (10)$$

where:

xk: Actual state vector of the system at the k-th step,

xref,k: The reference state vector that the system aims for in the k-th step,

Q: Diagonal matrix weighting the state error. In the study, diag[1200 1] was chosen.

R: A matrix or scalar value that serves to weight the control signal. In the study, the value was selected as 0.01.

2.3. Performance Metrics

In this study, various error and dynamic performance metrics are used to evaluate the system performance with PID control and Gradient-Based Model Predictive Control (MPC). ISE is an integral of squared errors over time, which weights more toward large errors. On the other hand, IAE computes an integral of absolute values of errors, indicating the overall magnitude of error. ITAE penalizes long-term errors by encouraging the system to reach the reference value more quickly. It contains information including Rise Time, the time the system needs to reach the desired value, and Settling Time, the time necessary for the response to stabilize. The overshoot is a measure of how much the system response exceeds the reference value, and Steady-State Error shows how accurately the system tracks the reference over time. These measurements provide the base for a thorough analysis of the steady-state and dynamic performance characteristics of the controllers.

2.4. Noise Characteristics

Random noise was introduced into the reference input in order to compare the performance of PID control and Gradient-Based Model Predictive Control (MPC) under noise. A Gaussian distribution with a zero mean and a standard deviation of σ=0.05 is the source of the noise. The system's input signal is represented as follows:

$$r_n(t) = r(t) + n(t) \quad (11)$$

Here, the reference signal is denoted by r(t), and the noise is denoted by n(t), which has the following definition:

$$n(t) \sim \aleph(0, \sigma^2) \quad (12)$$

The added noise was used to analyze the effects of the controllers on performance criteria such as steady-state accuracy, overflow and settling time.

3. Results and Discussion

In order to make a comparison for the control of the one-dof copter system, the responses of the PID controller with 2 different coefficients and the Gradient-based MPC controller against the standard step input were evaluated. The results obtained are given in Table 3.

Table 3: Performance metrics without noise

	PID-1	PID-2	Gradient-based MPC
ISE	0.74018	0.70948	1.2501
IAE	0.46023	0.47365	0.2763
ITAE	0.1122	0.5365	0.0665
Rise Time (s)	0.0827	0.12297	0.0801
Settling Time (s)	0.62051	0.21566	0.0981
Overshoot (%)	15.0072	1.0399	0.0
Steady-State Error (rad)	8.5256E-06	0.030004	0.0053

The error metrics were compared under noise-free conditions to evaluate the baseline performance of each controller. PID-2 outperforms PID-1 with a slightly lower value (0.70948 vs. 0.74018), indicating a reduced overall squared error. However, the Gradient-Based MPC has the highest ISE (1.2501), attributed to its focus on minimizing longer-term performance rather than instantaneous squared errors. Gradient-Based MPC shows the lowest absolute error (0.2763), reflecting its ability to track the reference trajectory more efficiently over time. Gradient-Based MPC significantly outperforms both PID controllers (0.0665 compared to 0.1122 and 0.5365), showcasing its robustness to time-weighted errors and superior performance in minimizing late-stage deviations.

The dynamic characteristics were analyzed under noise-free conditions to assess the inherent response behavior of each controller. Gradient-Based MPC achieves the fastest rise time (0.0801 s), followed closely by PID-1 (0.0827 s). PID-2 has a slower rise time (0.12297 s), likely due to its more conservative tuning to reduce overshoot. Gradient-Based MPC achieves the quickest settling time (0.0981 s), outperforming PID-2 (0.21566 s) and PID-1 (0.62051 s). This highlights MPC's advantage in rapidly stabilizing the system. Gradient-Based MPC eliminates overshoot entirely (0%), while PID-1 exhibits significant overshoot (15.0072%). PID-2 reduces overshoot to a minimal level (1.0399%), reflecting its design focus on stability. All controllers achieve near-zero steady-state error.

To evaluate the performance of the controllers under noise, simulations were performed using the same control parameters. The results obtained after this simulation are given in Table 4.

Table 4: Performance metrics with Gaussian noise

	PID-1	PID-2	Gradient-based MPC
ISE	0.75389	0.72422	1.2508
IAE	0.61008	0.52915	0.4802
ITAE	0.5638	0.67767	0.5166
Rise Time (s)	0.0827	0.12268	0.0807
Settling Time (s)	0.62346	0.21714	0.2122
Overshoot (%)	14.9631	1.3672	2.2812
Steady-State Error (rad)	0.0056969	0.030276	0.0013

Under the influence of Gaussian noise, all controllers exhibited some performance degradation compared to the noise-free case. Gradient-Based MPC maintained its robustness, particularly in minimizing long-term error accumulation, as evidenced by its relatively low ITAE even under noise. However, its IAE increased significantly, indicating a sensitivity to short-term disturbances. In contrast, PID-2 showed better noise resistance compared to PID-1, with improved stability and slightly better error metrics. Despite this, both PID controllers were more affected by noise than Gradient-Based MPC.

Dynamic characteristics such as rise time remained largely unaffected by noise across all controllers, showcasing their ability to maintain consistent initial responses. However, settling time and overshoot were more sensitive to noise. Gradient-Based MPC, which previously achieved zero overshoot, exhibited a small but noticeable increase, while PID-2 continued to outperform PID-1 in overshoot reduction and settling time stability.

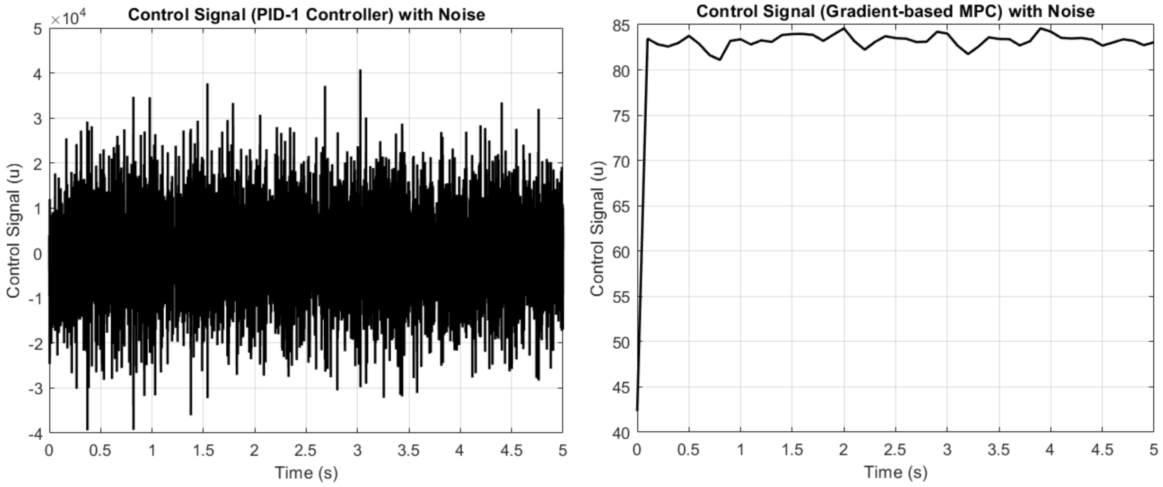
In terms of steady-state error, Gradient-Based MPC demonstrated superior noise rejection, achieving the smallest error under noisy conditions. PID-2 and PID-1 both experienced slight degradations in their steady-state performance, with PID-1 showing the largest sensitivity. Overall, Gradient-Based MPC remains the most robust choice under noisy environments, particularly for applications requiring precise steady-state accuracy and minimal long-term error. For systems with reasonable noise tolerance needs, PID-2 offers a more straightforward but reliable alternative.

One of the most remarkable differences existing between the two structures of controllers is the control signal. While the PID controllers produce a very noisy control signal, the control signal produced by the Gradient-Based MPC Controller method is much cleaner. Figure 3 illustrates the control signals of the controllers.

Gradient-Based MPC optimizes the control signal based on its predictions about the system's behavior in a number of subsequent phases. Future steps can more successfully eliminate noise and disruptions in the system thanks to this forecast. PID is more susceptible to noise because it simply corrects the current fault. MPC optimizes the control signal using a loss function. This function seeks to reduce the control signal's size as well as its inaccuracy. As a result, the control signal is considerably smoother and quieter.

In terms of producing smoother and quieter control signals, the gradient-based MPC controller outperforms PID controllers by a considerable margin. Its capacity to predict and optimize the control signal using a loss function that strikes a compromise between error reduction and signal amplitude is largely responsible for this. By accurately forecasting system behavior and minimizing disruptions, MPC provides a more dependable and efficient control method, particularly in noise-prone environments. Although PID controllers are still a simpler choice, their effectiveness in complicated or dynamic systems is limited by their increased sensitivity to noise and lack of predictive optimization.

The superior performance of Gradient-Based MPC in rise and settle times is due to the predictive nature of the method. MPC optimizes control signals by predicting the future behavior of the system. Thus, the system reaches the reference value quickly and can remain stable at this value. For example, although the settings of PID-1 and PID-2 controllers are optimized for shorter rise times, they lag behind MPC in metrics such as settling time and overshoot. The main reason for this is that PID controllers only respond to the current error and have limited impact on long-term performance. In contrast, MPC solves a cost function in each control loop, allowing the system to achieve both short-term and long-term goals more effectively.



(a) (b)

Figure 3: Control signals of Controllers under noisy input

4. Conclusions

This work has demonstrated the effectiveness of Gradient-Based Model Predictive Control (MPC) in controlling the dynamics of a one-degree-of-freedom (1 DOF) copter system by comparing its performance with conventional PID controllers. Gradient-Based MPC outperformed PID controllers in terms of accuracy and stability when noise was absent, reducing long-term errors, guaranteeing quicker settling times, and removing overshoot. In contrast to PID controllers, which showed more noise susceptibility and less smooth control signals, Gradient-Based MPC demonstrated greater robustness when exposed to Gaussian noise, maintaining superior steady-state accuracy and decreased sensitivity to disturbances.

Gradient-Based MPC's ability to produce smoother control signals highlights its promise for applications that demand stability and energy economy. Although PID controllers are straightforward and simple to use, their shortcomings in managing noise and dynamic constraints draw attention to the benefits of MPC's predictive optimization.

The findings indicate that Gradient-Based MPC is a more reliable and advanced solution for 1 DOF copter control, particularly in scenarios that require high precision and adaptability to environmental disturbances.

Further research could extend this comparison to multi-degree-of-freedom systems, thus providing additional validation of the scalability and efficacy of the proposed control strategies. The suggested Gradient-Based MPC's computing difficulties and practical effectiveness may be better understood by applying it to actual hardware systems, like autonomous aerial vehicles. Investigating adaptive tuning techniques for both PID and MPC controllers could improve their robustness in dynamically changing environments. Techniques such as reinforcement learning or model-free optimization could enable controllers to adjust their parameters in real time.

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