

Real-Time Speed Control of BLDC Motor Based On Fractional Sliding Mode Controller

Kamil Orman¹, Kaan Can², Abdullah Başçı², Adnan Derdiyok³

Accepted 3rd September 2016

Abstract: The design of the system used for brushless DC (BLDC) motor control in speed and position control is difficult due to the non-linear structure. Therefore, the designed controller is required to respond to these challenges and need high-efficiency operation. This paper presents the experimental validation of a robust speed control structure of a BLDC motor based on continuous sliding mode (CSM) and fractional-order sliding mode (FOSM) controllers. The controllers have been tested for low and medium speed reference signals and amplitude values. Then, both controllers have been compared in term of tracking performance and error elimination and the results have been shown graphically. Experimental results prove that the FOSM controller shows better trajectory tracking performance than CSM controller with high precision as well as good robustness against changes of references.

Keywords: BLDC motor, Continuous sliding mode control, Fractional sliding control, Speed control

1. Introduction

Parallel to the developments in the control areas, brushless DC motors are used in computers, automated office equipment, robotic applications, electro-mechanical systems and many precision machines. Brushless DC motors can be controlled more simply than other direct current motors and it has advantages such as high torque, high efficiency and small size. In addition, problems such as mechanical wear occur in the brushes and commutator by changing the position of the stator and rotor in the DC motor. And also maintenance of the brush takes a long time. But instead of brushes and commutator, BLDC motors use Hall Effect sensors [1]. Today, drivers have developed high processing ability and therefore robust control of BLDC motors are successfully carried out. The desired control algorithm is mathematically analysed for robust operation of the controller designed in accordance with, are tested in various computer programs and R & D work done. In the literature, various studies have been made for speed control of BLDC motors. In [2], is described a fuzzy logic approach for BLDC motor controller in variable speeds and the fuzzy logic tuner is used to adjust the gains of the PI controller and the results obtained in the simulation study showed less ripple under variation in system parameters with fast response times. Yu and Hwang have proposed an optimal PID controller and controller parameters are determined by linear quadratic regulator. The successes of the proposed method were compared with conventional PID controller and simulation and experimental results were given. Navidi et al. [4] proposed a method determined by ant colony search algorithm for PID controller parameters. They have demonstrated success with the simulation results of the proposed

method in improving the step response characteristics such as reducing the steady-states error, settling and rise time, and maximum overshoot in speed control. Chen and Tang proposed a sliding mode current control scheme for pulse width modulation (PWM) brushless dc motor drives in their study [5]. In this scheme, an improved “equivalent control” method is used and they stated that the validity of this scheme is achieved by simulation and experimental results. Moshiri et al. proposed an approach that has the merit to determine the optimal structure and the inference rules of fuzzy sliding mode controller simultaneously [6]. The success of the proposed controller is provided with the simulation results. Wang et al. offered a stable hierarchical sliding-mode control method for a class of second-order under actuated systems [7]. They consider the system as two subsystems and defined a first-level sliding surface for each part. Also they defined a second-level sliding surface for these two first-level sliding surfaces and simulation results have shown the success of the proposed method and adaptive abilities for all kinds of extraneous disturbances.

The main advantages of sliding mode control are demonstrated in numerous examples and simulations. The history of this control structure; In 1977, after V. Utkin [8] compiler work, Sliding Mode Controller - SMC and Variable Structure Control - VSC methods are widely used in control applications until today [9]-[19]. The aim of sliding mode control methods, especially in the real dynamic system; are designed to drive the system states onto a particular surface in the state space, named sliding surface. Once the sliding surface is reached, sliding mode control keeps the states on the close neighbourhood of the sliding surface. There are two main advantages of sliding mode control. First is that the dynamic behaviour of the system may be tailored by the particular choice of the sliding function. Secondly, the closed loop response becomes totally insensitive to some particular uncertainties. The disadvantage of sliding mode control method is a control signal that changes direction too much and it is called chattering. Also chattering causes some problems in practice such as damaging the component parts of the fast moving systems, to causes fatigue in the controlled system, significantly reduce the life of the system and to cause unnecessary energy consumption. Various methods are available to reduce this negative effect of chattering such as filtering, discontinuous approach, saturation function and fuzzy control. But this time the robustness of the

¹ Electronic and Automation Dept. Vocational High School, Erzincan University, Campus, 24000, Erzincan/Turkey

² Electrical and Electronics Engineering Dept. Engineering Faculty, Atatürk University, 25000, Erzurum/Turkey

³ Mechatronics Engineering Dept. Technology Faculty, Sakarya University, 54000, Sakarya/Turkey

* Corresponding Author: Email: korman@erzincan.edu.tr

Note: This paper has been presented at the 3rd International Conference on Advanced Technology & Sciences (ICAT'16) held in Konya (Turkey), September 01-03, 2016.

sliding mode control functionality is lost [12-14].

In this study, a fractional order sliding mode controller is designed to reduce the effect of chattering and also to maintain the high robustness and high accuracy features sliding mode control.

2. BLDC Motor Model

The electrical and mechanical mathematical equation of BLDC motor can be expressed as,

$$\frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = -\frac{R}{L_1} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} - \frac{1}{L_1} \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} + \frac{1}{L_1} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad (1)$$

$$\frac{d\omega_m}{dt} = \frac{1}{J_m} (T_e - B_m\omega_m - T_{Load}) \quad (2)$$

$$\varphi_e = \frac{p}{2} \varphi_m \quad (3)$$

$$\begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} = \begin{bmatrix} f_a(\varphi)\lambda\omega_m \\ f_b(\varphi)\lambda\omega_m \\ f_c(\varphi)\lambda\omega_m \end{bmatrix} \quad (4)$$

$$T_e = T_a + T_b + T_c \quad (5)$$

$$T_e = J_m \frac{d\omega_m}{dt} + B_m\omega_m + T_{Load} \quad (6)$$

$$T_{Load} = n^2 J_2 \frac{d^2\varphi_m}{dt^2} + n^2 B_2 \frac{d\varphi_m}{dt} + n F_c \frac{\varphi_m}{|\dot{\varphi}_m|} \quad (7)$$

where 'n' is the gearbox reduction ratio, 'B_m' is frictional coefficient of motor and load, 'J_m' is the motor inertia, 'J₂' is the gearbox inertia, 'F_c' is Coulomb torque constant, 'f_a(φ)', 'f_b(φ)', 'f_c(φ)' are functions having same shapes as back emfs, 'λ' is represent the total flux linkage as the product of number of turns and flux linkage/conductor, 'ω_m' is the angular speed of the motor, 'φ_m' is mechanical angle of rotor, 'φ_e' is electrical angle of rotor, 'p' is number of pole on rotor, L₁ = L - M, L is the self-inductance of the winding per phase, M is the mutual inductance per phase. T_{Load} is written in the (6) and it can be rearranged in the following form for each motor;

$$T_e = (J_m + n^2 J_2) \frac{d^2\varphi_m}{dt^2} + (B_m + n^2 B_2) \frac{d\varphi_m}{dt} + n F_c \text{sign}(\dot{\varphi}_m) \quad (8)$$

BLDC motor state equations are written in the following form;

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (9)$$

$$y(t) = Cx(t) \quad (10)$$

where the states and input vector are chosen as;

$$x(t) = [i_a i_b i_c \omega \varphi]^T \quad (11)$$

$$u(t) = [v_a v_b v_c T_{Load}]^T \quad (12)$$

the system matrices are given below,

$$A = \begin{bmatrix} -\frac{R}{L_1} & 0 & 0 & \frac{f_a(\varphi)\lambda}{J_m} & 0 \\ 0 & -\frac{R}{L_1} & 0 & \frac{f_b(\varphi)\lambda}{J_m} & 0 \\ 0 & 0 & -\frac{R}{L_1} & \frac{f_c(\varphi)\lambda}{J_m} & 0 \\ \frac{f_a(\varphi)\lambda}{J_m} & \frac{f_b(\varphi)\lambda}{J_m} & \frac{f_c(\varphi)\lambda}{J_m} & -\frac{B_m}{J_m} & 0 \\ 0 & 0 & 0 & \frac{p}{2} & 0 \end{bmatrix} \quad (13)$$

$$B = \begin{bmatrix} \frac{1}{L_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{L_1} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{L_1} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{J_m} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (14)$$

$$C = [0 \ 0 \ 0 \ 1 \ 0] \quad (15)$$

3. Control

In this section, the mathematical equations of designed controller and its block diagram is given.

3.1. Continuous Sliding Mode Controller

The goal is to drive states of the system given (9),(10) in the set S defined by;

$$S = \{x: \tau(t) - \xi(x) = \varepsilon(x, t) = 0\} \quad (16)$$

where τ(t) is the time dependent part of the sliding function, containing reference inputs to be applied to the controller. ξ(x) denotes the state dependent part of the sliding function, ε(x, t). The derivation of the control involves the selection of a Lyapunov function V(ε) and a desired form of derivative of the Lyapunov function such that closed-loop system is stable. The selected Lyapunov function is [20-22]

$$V = \frac{1}{2} \varepsilon^T \varepsilon \quad (17)$$

which is positive definite, and its derivative is

$$\dot{V} = \varepsilon^T \dot{\varepsilon} \quad (18)$$

The solution ε(x, t) = 0 will be stable if time derivative of the Lyapunov function can be expressed as [22]

$$\dot{V} = -\varepsilon^T D \varepsilon \quad (19)$$

where D is a positive definite matrix. Thus, the derivative of the Lyapunov function will be negative definite and this will ensure the stability. Eq. (18) and (19) lead to

$$\varepsilon^T (D\varepsilon + \dot{\varepsilon}) = 0 \quad (20)$$

A solution for this equation is

$$D\varepsilon + \dot{\varepsilon} = 0 \quad (21)$$

The expression for derivative of the sliding function is

$$\frac{d}{dt} \varepsilon = \frac{d}{dt} \tau - \frac{d}{dt} \xi \quad (22)$$

where,

$$\xi = Gx(t) \quad (23)$$

$G \in \mathbb{R}^{n \times m}$ is gain matrices, and

$$\dot{\xi} = G\dot{x}(t) \quad (24)$$

First, equivalent control is found by $\dot{\varepsilon} = 0$ and using (22) as

$$\dot{\varepsilon} = \dot{\tau} - \dot{\xi} = \dot{\tau} - (GAx(t) + GBu_{eq}) = 0 \quad (25)$$

$$u_{eq} = (GB)^{-1}(\dot{\tau} - GAx(t)) \quad (26)$$

Second, using (23) the control input to the system can be found by following:

$$\dot{\varepsilon} = -D\varepsilon = \dot{\tau} - \dot{\xi} \quad (27)$$

$$\dot{\tau} - (GAx(t) + GBu_{eq}) = -D\varepsilon \quad (28)$$

and the result of the short algebra can be written as

$$u = u_{eq} + (GB)^{-1}D\varepsilon \quad (29)$$

Third, from time derivative of the sliding function

$$\dot{\varepsilon} = \dot{\tau} - (GAx(t) + GBu) \quad (30)$$

multiplying both sides with $(GB)^{-1}$

$$(GB)^{-1}\dot{\varepsilon} = (GB)^{-1}(\dot{\tau} - (GAx(t) + GBu)) \quad (31)$$

and by using (25)

$$(GB)^{-1}\dot{\varepsilon} = u_{eq} - u \quad (32)$$

and finally when this equation is substituted in (29) the control is found as

$$u(t) = u(t^-) + (GB)^{-1}(\dot{\varepsilon} + D\varepsilon) \quad (33)$$

$$t = t^- + \Delta, \Delta \rightarrow 0$$

The value of the control at the instant t is calculated from the value at the time $t^- + \Delta$ and the weighed sum of the control error ε and its time derivative. Control (33) is continuous function everywhere except in the points of discontinuity of the function $\varepsilon(x, t)$. When these equations are adapted for BLDC motor control system shown in Fig. 1, the following equation can be written for the control loop as

$$u_v(t) = u_v(t^-) + (GB)^{-1}(\dot{\varepsilon}_v + D\varepsilon_v) \quad (34)$$

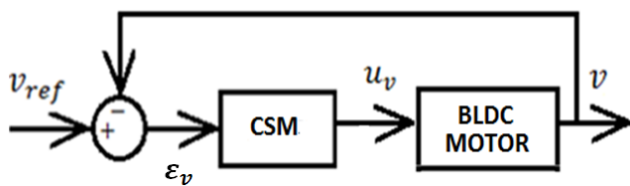


Figure 1. Continuous sliding mode controller block diagram

3.2. Continuous Sliding Mode Controller

The fractional-order differentiator can be denoted by a general fundamental operator ${}_a D_t^p$, where a and t are the limits of operations. The fractional-order differentiator and integral are defined as follows,

$${}_a D_t^p = \begin{cases} \frac{d^p}{dt^p} & : p > 0 \\ 1 & : p = 0 \\ \int_a^t (d\tau)^{-p} & : p < 0 \end{cases} \quad (35)$$

where p is the fractional order which can be a complex number, however the constant p is related to initial conditions. There are several mathematical definitions to describe the fractional derivatives and integrals [23], [24]. Between these definitions, here are two commonly used ones, i.e., the Grünwald–Letnikov (GL) and the Riemann–Liouville (RL). The GL definition is;

$${}_a D_t^p f(t) = \lim_{h \rightarrow 0} h^{-p} \sum_{j=0}^{\lfloor \frac{t-a}{h} \rfloor} (-1)^j \binom{p}{j} f(t - jh) \quad (36)$$

where $[\cdot]$ means the integer part, while the RL definition is given as,

$${}_a D_t^p f(t) = \frac{1}{\Gamma(n-p)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{p-n+1}} d\tau \quad (37)$$

for $(n-1 < p < n)$, $\Gamma(\cdot)$ is the Euler's gamma function, a is the initial time and t parameter is used when the differential and integral are taken.

3.3. Fractional-Order Sliding Mode Controller

In (34), if the derivative term expressed as a fractional order,

$$u_v(t) = u_v(t^-) + (GB)^{-1}({}_a D_t^p \varepsilon_v + D\varepsilon_v) \quad (38)$$

When these equations are adapted for BLDC motor control system shown in Fig. 2

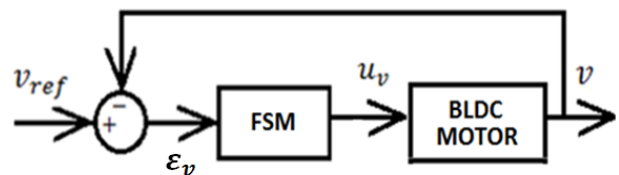


Figure 2. Fractional sliding mode controller block diagram

4. Experimental Result

The performance of the both controllers are evaluated for the tracking capability of the references and the ability to reduce the error and rise time. Sinus and trapezoidal speed references are implemented to BLDC motor for performance comparison of both controllers and the experimental results are shown in Figs. 3-10.

In the first experiment, 10 rpm sinusoidal speed reference is chosen for testing CSM and FOSM controllers. As shown in Fig. 3 and Fig. 6, the FOSM controller has fast rise time at the reference input in comparison with CSM controller. Although both controller have similar reference tracking capabilities and it is seen that the CSM controller is less sensitive to follow the reference. The maximum percentage errors of sine wave reference for CSM is 10.13% and for FOSM 4.17 % respectively.

The results for sinusoidal speed reference at 1000 rpm are given in Fig. 4 and Fig. 7. Due to chosen a reference signal which is slowly changing over time the tracking errors are close to each other. However, FOSM gives 0.4 sec rise time while CSM is 0.6 sec which is obviously much better.

In the second experiment, trapezoidal wave speed reference at 10 rpm is given for control of BLDC motor. The trapezoidal wave reference is important to test the performance of the controllers for sudden changes. It can be seen from Fig. 5 and Fig. 8 that, CSM gives 0.6 sec rise time while CSM is 0.5 sec.

On the other hand, FOSM have better performance than CSM when the trapezoidal wave reference sudden changes. The trapezoidal speed reference at 1000 rpm in Fig. 9 and Fig. 10 also shows the success of the FOSM. Both controllers have similar reference tracking capabilities and have similar rise time at the starting point of trapezoidal wave. CSM was failed to show adequate performance at the moments of suddenly changes. FOSM has been more successful than CSM at the any sudden change after the start.

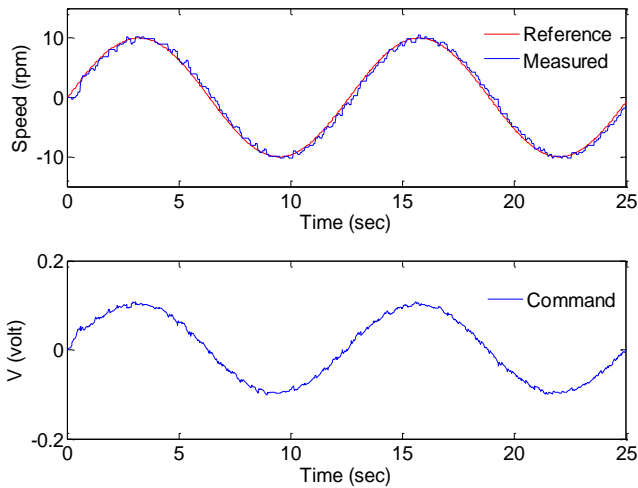


Figure 3. Continuous sliding mode 10 rpm sinus ref.

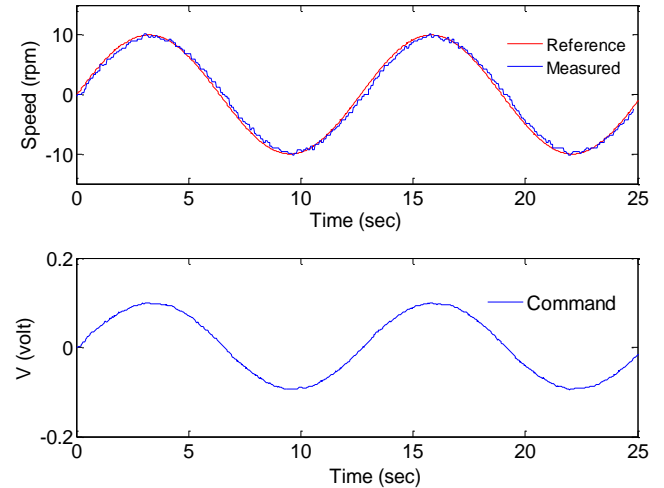


Figure 6. Fractional-order sliding mode 10 rpm sinus ref.

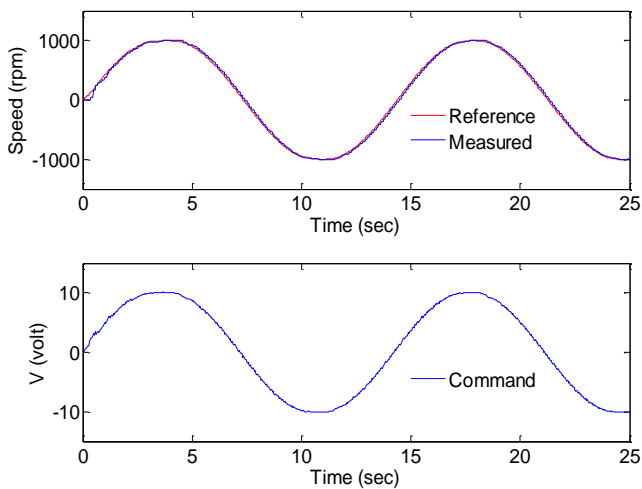


Figure 4. Continuous sliding mode 1000 rpm sinus ref.

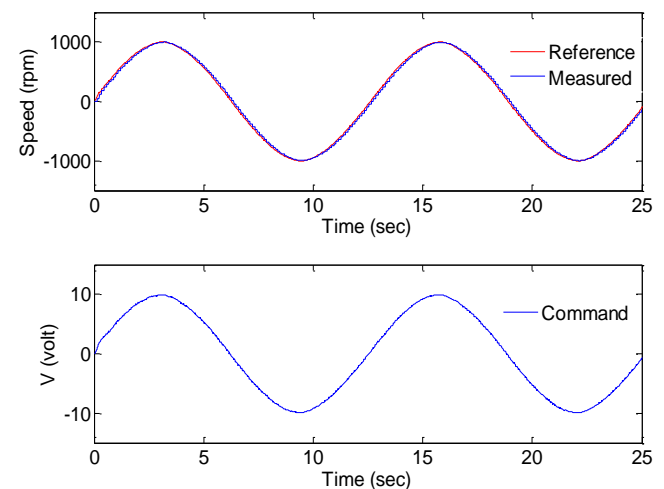


Figure 7. Fractional-order sliding mode 1000 rpm sinus ref.

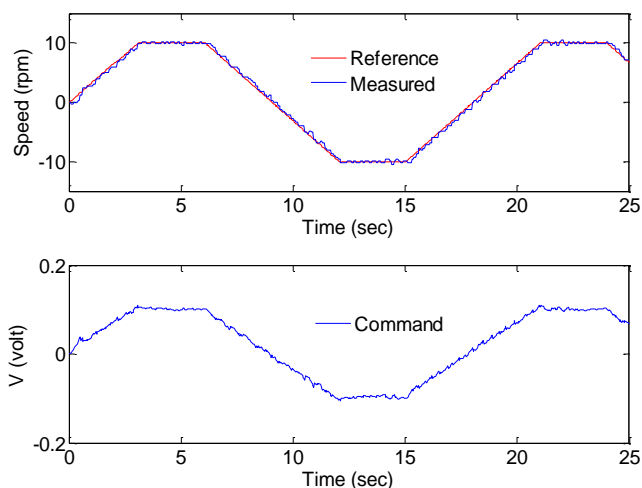


Figure 5. Continuous sliding mode 10 rpm trapezoidal ref.

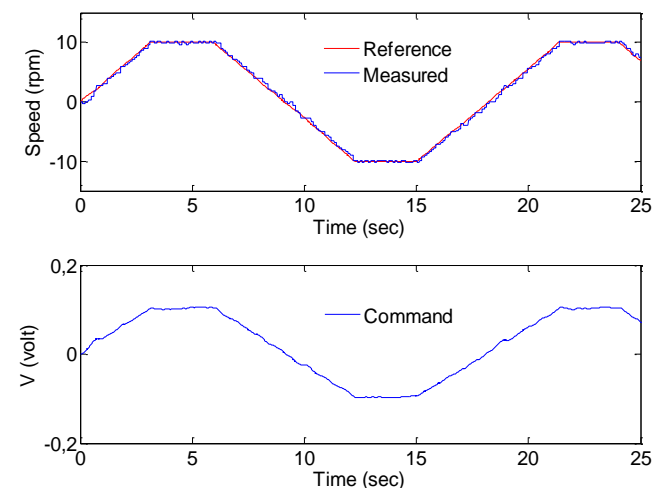


Figure 8. Fractional-order sliding mode 10 rpm trapezoidal ref.

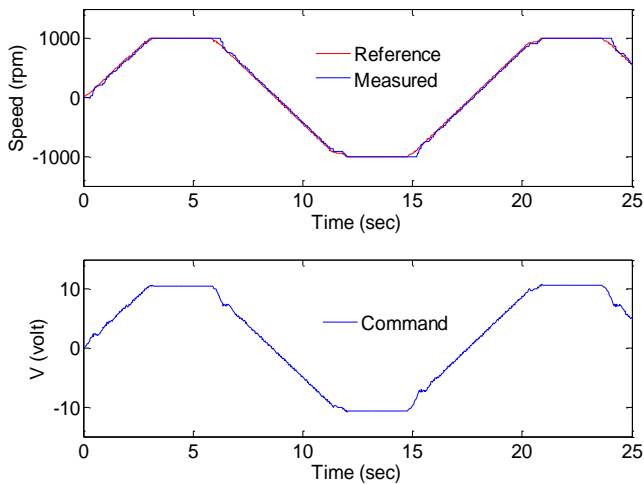


Figure 9. Continuous sliding mode 1000 rpm trapezoidal ref.

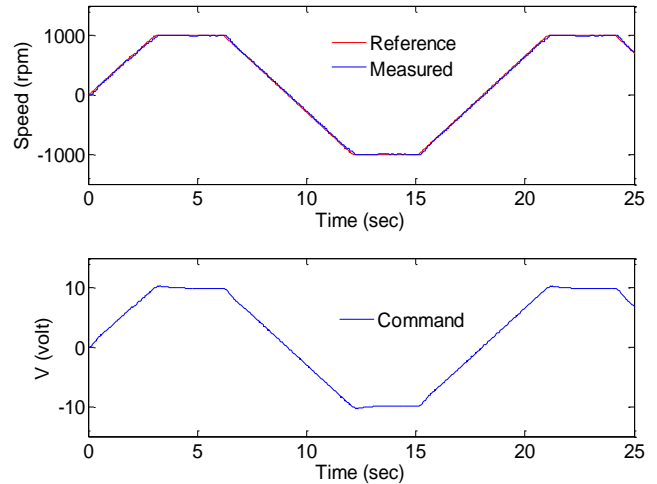


Figure 10. Fractional-order sliding mode 1000 rpm trapezoidal ref.

5. Conclusions

In this paper, an experimental study on the application of CSM and FOSM controllers to a BLDC motor under the different speed references was presented. The experimental results show that the FOSM controller shows better steady state performance with better rise time, smaller speed error and having less overshoot when it compared to the responses of CSM. To conclude, the applied FOSM controller results in better responses than CSM controller to control the speed of the BLDC motor under changing references.

References

- [1] Allan R. Hambley, *Electrical Engineering: Principles and Application*, Prentice Hall, New Jersey 1997.
- [2] Lee, C. K., and W. H. Pang. "A brushless DC motor speed control system using fuzzy rules," *Power Electronics and Variable-Speed Drives. Fifth International Conference on. IET*, pp. 101-106, 1994.
- [3] Yu, G-R and Hwang, R-C. "Optimal PID speed control of brush less DC motors using LQR approach," In: *Systems, Man and Cybernetics, 2004 IEEE International Conference on. IEEE, 2004. p. 473-478.*
- [4] Navidi, N., M. Bavafa, and S. Hesami. "A new approach for designing of PID controller for a linear brushless DC motor with using ant colony search algorithm," *2009 Asia-Pacific Power and Energy Engineering Conf.e. IEEE*, pp.1-5, 2009.
- [5] Chen, Jessen, and Pei-Chong Tang. "A sliding mode current control scheme for PWM brushless DC motor drives," *IEEE transactions on Power Electronics* 14.3 pp. 541-551, 1999.
- [6] B. Moshiri, M. Jalili-Kharaajoo, F. Besharati, "Application of fuzzy sliding mode based on genetic algorithms to control of robotic manipulators", *Emerging Technologies and Factory Automation*, Vol. 2, pp. 169 – 172, 2003.
- [7] Wang, W., et al. "Design of a stable sliding-mode controller for a class of second-order underactuated systems." *IEE Proceedings-Control Theory and Applications* 151.6, pp. 683-690, 2004.
- [8] Vadim, I. Utkin. "Survey paper variable structure systems with sliding modes," *IEEE Transactions on Automatic control* 22.2, pp. 212-222, 1977.
- [9] Derdiyok, A., Guven, M. K., Inanc, N., Rehman, H. U., & Xu, L. "A DSP-based indirect field oriented induction machine control by using chattering-free sliding mode," In *National Aerospace and Electronics Conf., NAECON 2000. pp. 568-573, 2000.*
- [10] Nguyen, D., *Sliding-Mode Control: Advanced Design Techniques*, Phd Thesis, University of Technology, Sydney, 1998.
- [11] Hung, J.Y., Gao, W., ve Hung, J.C. "Variable structure control: A survey", *IEEE Transactions on Industrial Elect.*, Vol 40, No 1, 2–22, 1993.
- [12] Eker, İ., "Sliding Mode Control with PID Sliding Surface and Experimental Application to An Electromechanical Plant", *ISA Transactions*, vol.45, pp.109-118, Number 1, January 2006.
- [13] Özdal, O., "Model Dayanaklı Kayan Kipli Denetim", Master Thesis, Hacettepe Üniversitesi FBE, Ankara, 2008.
- [14] Kızmaz, H., "Asılı Sarkacın Kayma Kipli Kontrolü", Master Thesis, SAU FBE, Sakarya, June 2009.
- [15] R. Benayache, L. Chrifi-Alaoui, P. Bussy and J.M. Castelain, "Design and implementation of sliding mode controller with varying boundary layer for a coupled tanks system". *17thMediterranean Conference on Cont. & Aut.*, p:1215-1220, 2009.
- [16] A. Levant, "Chattering Analysis," *IEEE Transactions on Automatic Control*, Vol.55, pp. 1380-1389, 2010.
- [17] T. Floquet, S. K. Spurgeon and C. Edwards, "An Output Feedback Sliding Mode Control Strategy for MIMO Systems of Arbitrary Relative Degree," *International Journal of Robust and Nonlinear Control*, Vol. 21, No. 2, 2010.
- [18] Derdiyok, A. and Başçi, A., "The application of chattering-free sliding mode controller in coupled tank liquid-level control system," *Korean Journal of Chemical Engineering*, 30(3), pp.540-545, 2013.
- [19] Soysal, Birol. "Real-time control of an automated guided vehicle using a continuous mode of sliding mode control." *Turkish Journal of Electrical Engineering & Computer Sciences* 22.5, pp.1298-1306, 2014.
- [20] K. Jezernik, M. Rodic, R. Safaric and B. Curk, B., "Neural network sliding mode robot control," *Robotica*, 15(1), pp. 23-30, 1997.
- [21] A. Sabanovic, K. Jezernik and K. Wada," Chattering-free sliding modes in robotic manipulators control." *Robotica*, 14, 17, 1996.
- [22] A. Derdiyok and M. Levent, "Sliding mode control of a bioreactor." *Korean J. Chem. Eng.*, 17(6), 619, 2000.
- [23] I. Podlubny, *Fractional Differential Equations*, Academic Press, San Diego, California, 1999.
- [24] I. Podlubny, "Fractional-order systems and PI λ D μ controllers", *IEEE Transactions on Automatic Control*, vol. 44(1), pp. 208–214, 1999.