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A Comparison of Current and Alternative Production Characteristics of a Flow Line: Case Study in a Yarn Producer's Packaging Unit

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Abstract: Production lines are consecutively placed machines designed to obtain short cycle times with high speeds. This type of flow line is preferred when the demand pattern occurs in high volumes from the same product in short production periods. The structure of production systems is directly related to the quantity and variety of the demand. If the overall demand is made up of an identical product in high amounts in a short period of time, flow lines are designed to answer this need in a manner of consecutive linear machines, capable of performing one or more tasks per machine. Production with low cost and right quantity conditions is also an obligation under timely constraints. A packaging station of a five machine Bernoulli line is modelled in this paper. Two alternative packaging materials are put into consideration against a readily used material and those 3 packaging films are compared according to the performance characteristics. A C# programme is coded to obtain the statistical performance characteristics of an aggregation method applied to the "Bernoulli flow line" to make a decision on which material is to be selected. Production rates, blockages, starvations as well as work in process stocks are the performance values calculated by the C# code developed, according to an aggregation method. One of the two competing alternatives is selected after analyzing the outcomes of the software.

Keywords: *Automatic production systems, maintenance and repair times, inventory management*

1. Introduction

Analysis on "single production machines" and "production lines" are two important topics of industrial engineering discipline. Both, the nature of the incoming entities and the analysis of the operations performed on those production lines can be applied by using a lot of different techniques. From scheduling of the arrivals to line balancing, numerous approaches exist subject to further researches and they have been studied for decades as well. This paper will include the analysis of consecutive machines of a production line under given performance criteria. Performance calculations of various operations in a case study showing Bernoulli characteristics are carried out and the main procedure followed shows concordance with the Bernoulli production line model covered in detail in related literature [1].

2. Literature Review

Theoretical studies have been applied to production lines for a long time. From a broader view, stable and instable systems are under the radar of the researchers for 50 years. The most dominant part of these studies are being stable systems [2, 3].
The nature of the entity arrivals to the assembly lines, Kanban systems [4], Markovian queue systems [4], flow, transfer and assembly processes of production systems [2, 3, 5], can be counted under this title. On the other hand, the researches on "unstable systems" can hardly be told to be much in quantity. Somehow those researches are very valuable and seem to be promising for the future works [1, 6].
The bottlenecks in production and assembly are also another topic of interest [7] and lots of studies were applied to determine the effects of;

- Downstream and upstream machines on output quantity
- The bottleneck creation effects of materials under process.

As an addition to "job shop" and "production line" type processes, re-entrant production type has also been defined and lots of studies are performed on this type of production. The parts visiting the same machines more than once are the main concern in this type [5, 8, 9]. Performance criteria such as, WIP, failures, production rate etc. are also studied in these papers. The process modelling with buffers and size of them are studied in detail in many papers. Topics like production rate, machine stoppages are highly emphasized in these studies.
Fernandes et. al. included and emphasized the assumptions and working principles of Bernoulli production lines and provides a basis for further researches to be carried out. According to the work: The exact solution of small unreliable lines was introduced by numerous papers [11-13]. Phase-type modelling about production lines was presented in 1985 [14]. Heavey et. al. discusses a highly efficient numerical analysis that calculates the transition matrix to assess the prior works [15].There is a need for computational procedure in large production lines since there are large number of Markov chain states of the systems. Hillier and So, introduced a method for solving reliable Erlang and exponential production lines [16]. As a further extension phase-type modelling in mixed generalized exponential distributions is also applied and presented in [3]. Gershwin showed a

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decomposition method for the approximate evaluation of tandem queues with finite intermediate buffers and blocking [17].

Tolio et al have given a decomposition approach for the performance of evaluation of automated flow lines having multiple failure modes [18]. An analytical approach was pointed out for performance evaluation of an automated flow line which includes the dependency between the production and the repair system in [19]. An aggregation method for analysis of flow lines was developed in [20]. Patchong and Willaeys studied sets of parallel machines which are replaced by an equivalent single representative machine [21]. By the help of this logic, large lines with multiple parallel machines were analyzed from performance measure point of view in an easier manner. Similar works to this logic also exist in [22-24]. Researchers in [25-27] developed efficient decomposition methods with exponential processing times, multiple failure modes and finite buffer capacities. Nonlinear flow lines are also analyzed in the literature [28-30]. Exponentially distributed repair and failure times are covered in [30-31]. SAN (Stochastic Automata Network) and Markovian modeling are studied in this context [35].

Bernoulli production lines are studied from different point of views in numerous papers [7, 20, 33, 34]. Like Buzacott, many authors have studied the performance criteria such as WIP, failures, production rates, efficiency etc. of the production lines [35].

In the case called “buffer allocation problem”, the marginal surplus of (n+1)th work in process (WIP) unit is compared with different parameters (space, cost etc.) while n unit of WIP is available. Even though the additional unit in the “work in process” stocks decreases, the dependency of the upstream machine to the prior one increases. The required space or the cost of making this adjustment in the production system may need complex calculations. This process may be held relatively easily if the system includes 2 machines, but as the size of the system increases, calculations may need too much time and calculations.

3. Literature Review

This study is prepared in the packaging department of a yarn manufacturer located in Uşak-Turkey. Process is made up of five serial machines showing Bernoulli characteristics. It is planned to test two alternative materials and the current material under use, to choose the option that gives the best results from performance criteria point of view. A C# code is developed to make the necessary calculations and these results are shown to be verified by the PSE Toolbox software [1].

The basic assumptions are given below:

- Environmental parameters were set to be same for the productions done with both current and alternative materials. During production runs, no other materials but only the chosen material is used not to effect the individual material performances.
- During the three production runs no machine was subtracted or added to the process.
- All machines have equal service times.
- Transport times between the stages are negligible.
- Machines are starved when their upstream buffers are empty and machines are blocked when their downstream buffers are full.
- All the upstream machines can increase the WIP stock of their next buffer by “one” unit meanwhile the downstream machines can only reduce the buffer size by “one” unit.
- The status of each machine is determined independently from the other machines.
- First machine is never starved due to excess input amount.
- Last machine is never blocked since it has enough area for the outputs.

3.1. Bernoulli Distribution

Bernoulli Distribution is the probability distribution of a random variable that can only take two possible values, 0 or 1. When it takes the value 1, then p =1 and from q=1-p, q becomes 0. This 0-1 binary structure is also valid for yes-no fail-work, false-true etc. An experiment which can have only two outcomes is called a “Bernoulli experiment”. In these experiments p is the probability of success and q=1-p becomes the probability of failure. Bernoulli experiments are repeatable and they can have just two outcomes. The probability of success cannot change from one experiment to the other. Each experiment is independent from the others.

S = { x / 0,1 }

The probability mass function f of Bernoulli Distribution is:

$$f(x; p) = p^{x}(1-p)^{1-x} \quad x=0,1 \tag{1}$$

The mean and the variance are as follows:

$$\mu = E(x) = p \tag{2}$$

$$\sigma^2 = Var(x) = p(1-p) = pq \tag{3}$$

3.2. Structure And The General Properties Of The Bernoulli Lines

A “Bernoulli line” is a production line performing repetitive production tasks with serial machines which are working with identical cycle times (Figure 1). Failures are time or operation dependent [27, 36]. This paper includes a case study that there is a constant failure probability for all the N machines in the given cycle time.

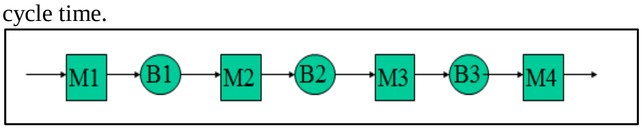


Fig. 1. Bernoulli Production Line

In a Bernoulli Line, machines will be active with p and will not with (1-p) probabilities.

Bernoulli lines can be symbolized by using a vector which is given below as (p_1, ..., p_M, N_1, ..., N_{M-1}).

A machine is UP with probability (p). Buffer capacity (N) and probability values are shown as the parameters in the vector.

The ratio of active state of a machine is shown with (pi) and this statistics is independent of the machines’ historical data. This property is called “the memoryless property”.

Cycle time of the machines is denoted with τ and is given as:

$$\tau = \min\{\tau_i, \forall i\} \tag{4}$$

$$p_i = \tau e_i / \tau_i \text{ve } e_i = 1/(1 + \lambda/\mu_i) \tag{5}$$

$$N_i = \min\{h_i \mu_i \tau_{i+1}, h_i \mu_{i+1} \tau_i\} + 1 \tag{6}$$

Meerkov and Li show the required calculations of the characteristics of "two and more machines" of Bernoulli production lines [1]. Machines will be blocked (even the machines themselves are not failed) when their downstream buffer is full and the downstream machine is not active. Machines are starved when they are ready to produce but their upstream buffers are empty. As a result, first machine is never starved (when there is excess input amount) and last machine is never blocked (output depot volume is assumed to have infinite capacity).

Memoryless property is a characteristic of the Bernoulli machines. System states concur with the states of the buffer. As the dimension of the systems start from 2 and goes to n, the complexity of the overall system increases exponentially.

Buffer *i* and the system have (N_i +1) and (N_1+1)(N_2+1)……..(N_{M-1}+1) states respectively. Since a direct analysis of such a complex system wouldn't be practical, an aggregation approach should be used [1].

3.3. Aggregation

Bernoulli parameters p_i^b (production rate of backward aggregation) and p_i^f (production rate of forward aggregation) are assigned as the "production rate" of the aggregated "two-machine line". Let M be the length of the line, if M-th and M-1 th machines are aggregated into a single Bernoulli machine (m_{M-1}^b) and the process is continued until the first machine then all machines are aggregated into $m_1^b$where b stands for backward aggregation. However the backward calculation and the production rate of M machine line may not coincide. A forward aggregation is defined to remedy this problem a forward aggregation is defined. The first machine (m1) is aggregated with the backward aggregated rest of the line(m_2^b). This aggregation is continued until the last machine and (m_M^f) is found where f signifies forward aggregation. Again m_M^f which is the Bernoulli parameter may not coincide with the line production rate similar to the backward aggregation. Forward and backward aggregation procedures are shown in Figures 2 and 3.

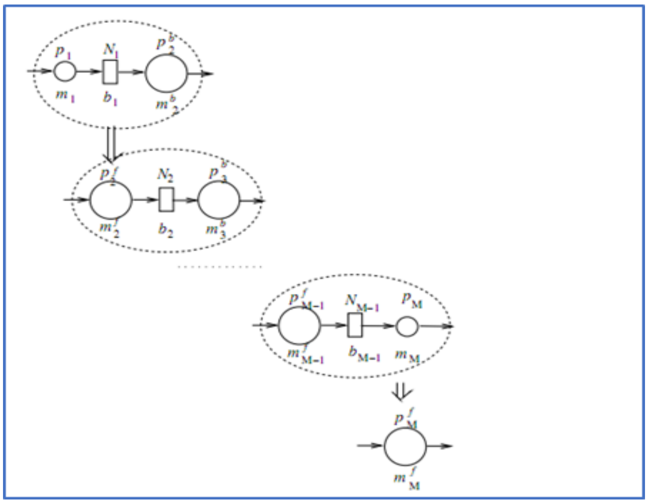


Fig. 2. Forward Aggregation [1]

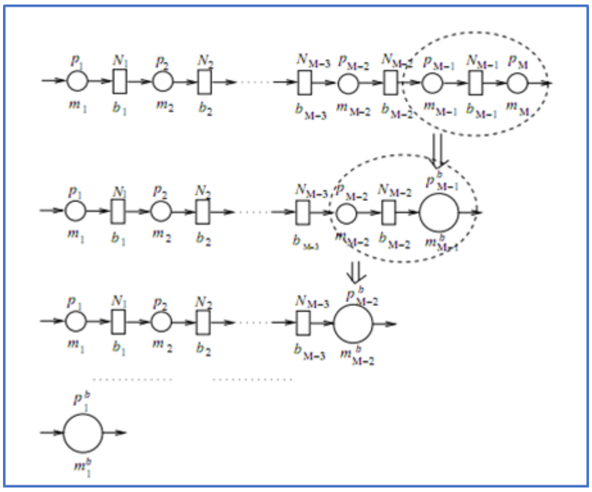


Fig. 3. - Backward Aggregation [1]

3.4. Recursive Aggregation

To alleviate the discrepancy an iteration procedure is introduced between backward and forward aggregations. Recursive aggregation notation is given below:

$$p_i^b(s+1) = p_i[1 - Q(\,p_{i+1}^b(s+1), p_i^f(s), N_i)]$$
$$i = 1, \ldots, M-1 \qquad (7)$$

$$p_i^f(s+1) = p_i[1 - Q(\,p_{i-1}^f(s+1), p_i^b(s+1), N_{i-1})]$$
$$i = 2, \ldots, M \qquad s = 0,1,2, \ldots, \qquad (8)$$

$$p_i^f(0) = p_i \qquad i = 1, \ldots, M \qquad (9)$$

$$p_i^f(s)p_1\,, \qquad s, \qquad (10)$$

$$p_M^b(s) = p_M\,, \qquad s = 0,1,2, \ldots, \qquad (11)$$

$$Q(x,y,N) = \begin{cases} \dfrac{(1-x)(1-\propto)}{1-\dfrac{x}{y}\propto^N}, & \text{if } x \neq y \\ \dfrac{1-x}{N+1-x}\,, & \text{if } x = y \end{cases} \qquad (12)$$

And;

$$\propto = \frac{x(1-y)}{y(1-x)} \qquad (13)$$

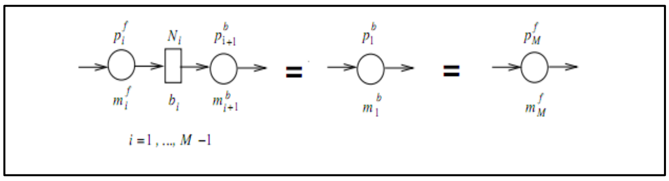


Fig. 4. An Illustration of the Aggregation Procedure [1]

M-machine long systems can be illustrated with a single representative machine by using recursive forward and backward aggregation procedure. As soon as the procedure is over, the study of performance criteria analysis follows. *Production Rate* (PR_i) is the average quantity of outputs processed by the final machine of an assembly line in a cycle time"[1].

$$\widehat{PR} = p_1^b = p_M^f \qquad (14)$$
$$= p_{i+1}^b[1 - Q(p_i^f, p_{i+1}^b, N_i)] \qquad (15)$$
$$= p_i^f[1 - Q(p_{i+1}^b, p_i^f, N_i)] \qquad (16)$$
$$i = 1, \ldots, M-1$$

When $p_1^b = p_M^f$ holds, "Production Rate" is accepted to be reached. The average number of parts in the i- th buffer of a production system in the steady state is called the work in process of the i-th buffer [1]. ($\widehat{WIP_i}$) is defined as:

$$\widehat{WIP} = \begin{cases} \dfrac{p_i^f}{p_{i+1}^b - p_i^f \propto^{N_i}(p_i^f, p_{i+1}^b)} * \left(\left[\dfrac{1-\propto^{N_i}(p_i^f, p_{i+1}^b)}{1-\propto(p_i^f, p_{i+1}^b)}\right] - [N_i \propto^{N_i}(p_i^f, p_{i+1}^b)]\right), & \text{if } p_i^f \neq p_{i+1}^b \\ \dfrac{N_i(N_i+1)}{2(N_i+1-p_i^f)}, & \text{if } p_i^f = p_{i+1}^b \end{cases} \qquad (17)$$
$$i = 1, \ldots, M-1$$

Blockage of machine *i* (BL_i)is the steady state probability that machine *i* is up, buffer *i* is full, and machine *(i+1)* can not take a part from its downstream buffer.
Starvation of machine *i* (ST_i) is the steady state probability that machine *i* is up and its upstream buffer *(i-1)* is empty
Estimates of $\widehat{BL_i}$ and $\widehat{ST_i}$ are given below:

$$\widehat{BL_i} = p_i Q(p_{i+1}^b, p_i^f, N_i), \qquad i = 1, \ldots, M-1 \qquad (18)$$
$$\widehat{ST_i} = p_i Q(p_{i-1}^f, p_i^b, N_{i-1}), \qquad i = 2, \ldots, M \qquad (19)$$

A simple model of a Bernoulli Production Line of two machines is given in Fig.5:

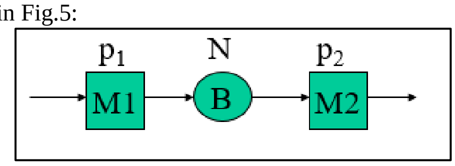


Fig. 5. A Bernoulli Production Line of Two-Machine

Production rate of serial consecutive machines is calculated in "backward aggregation procedure" as follows:

$$p_i^b = p_i[1 - Q(p_{i+1}^b, p_i^f, N_i)] \qquad (20)$$
$$= p_i - \widehat{BL_i} \ ,$$

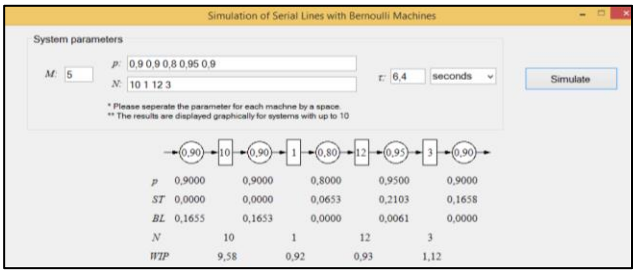
Production rate of serial consecutive machines is calculated in "forward aggregation procedure" as follows:

$$p_i^f = p_i[1 - Q(p_{i-1}^f, p_i^b, N_{i-1},)] \qquad (21)$$
$$= p_i - \widehat{ST_i}$$

3.5. C# and PSEToolbox Programs

Calculations of forward and backward aggregation are time consuming and complex. Hence, a computer programme is highly needed. PSEToolbox is a programme developed for this purpose [1]. Programme is designed to be able to work in demo mode if the commercial version is not activated. PSEToolbox finds the system characteristics as far as the production line does not exceed 5 machine long in demo mode. By taking into account that a line may exceed 5 machines, a C# code is prepared for this paper. C# outputs are compared and verified with the PSEToolbox results. PSEToolbox data entry panel is shown in Fig. 6.

Fig. 6. Data Entry to PSEToolbox



C# code allows the users obtain the results of production lines having more than five machines. Code applies the aggregate algorithm [1] and gives the performance criteria of the line.
A comparison between the C# code and the PSEToolbox is made and the outputs are found to be in accordance with each other.
Outputs of the program show that, as the iteration number progresses the p_i^b ve p_i^f values approach and finally bind on each other. Afteron, the value which equalizes pf and pb is called as "production rate"($\widehat{PR}$). At the following stages, the C# programme calculates the performance criteria; work in process inventory ($\widehat{WIP_i}$), starvations($\widehat{ST_i}$) and blockages($\widehat{BL_i}$).
Programme is a C# DOS command code and the outputs need to be passed to a user friendly environment. Due to this need, outputs are printed to MS Excel output. By making the proper adjustments, these outputs can easily be graphed or listed according to the analyst's needs.

3.6. Outputs and The Explanation of the Program

The real life data is entered to both PSEToolbox and C# programs and the outputs called as "performance criteria" are obtained. Performance criteria called as production rate ($\widehat{PR}$) work in process amount ($\widehat{WIP_i}$), starvations ($\widehat{ST_i}$)and blockages ($\widehat{BL_i}$)are all calculated according to aggregate algorithm (1) by both of the programs. Calculations are done for current and two alternative cases and discussion on the performance of these 3 cases are presented in the final part.

4. Application

4.1. Modelling of Current and Alternative Cases

The process of packaging machines given in this paper is located in a yarn manufacturer's packaging department in Uşak-Turkey. Machines in the line are set serially and carry the characteristics of a Bernoulli production line. Packaging production line consists of the following stages:

- Packaging : Every single cone is covered with nylon
- Packaging 2 : Single cones are covered with another nylon and taken to an upper platform with a mechanism.
- Shrinking :12 cones are shrinked and taken into a bag
- Cooling : Bags are cooled off .
- Weighing : Bags are weighed automatically and got ready for sewing and transport.

As the circles symbolizes the processes and the rectangles the buffers, the five machine illustration of the packaging line is shown on Fig. 7.The busy state ratio (p values) of the machines are presented on Table 1.

Packaging dpt works with a certain material for a long time but two alternative materials are also members to be used in the department. To choose the best option for the process, a test production is done using both of the alternative materials, the p values are obtained and written down on Table 1.

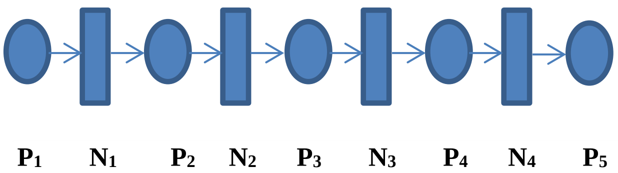


Fig. 7 - Petri Net modelling of the Current Bernoulli Production Line

Table 1. p values

p ratio	Machine 1	Machine 2	Machine 3	Machine 4	Machine 5
Current Material	0,90	0,90	0,80	0,95	0,90
Alt. Mat. 1	0,8	0,8	0,75	0,85	0,8
Alt. Mat. 2	0,95	0,95	0,95	0,95	0,95

Capacities (N) of the buffers between the machines are set to be equal (shown in Table 2) for all the materials.

Table 2. Max Buffer Capacities For Current and Alternative Cases

Buffers	**Buffer 1**	**Buffer 2**	**Buffer 3**	**Buffer 4**
Current Material	10	1	12	3
Alternative Material 1	10	1	12	3
Alternative Material 2	10	1	12	3

4.2. Performance Criteria

4.2.1. Production Rates:

pf and pb values are the first values to be found in aggregate method (1). These values are calculated by the C# programme presented on Table 3:

Table 3. Forward (PF) and Backward (PB) Aggregation for Current and Alternative Cases

Itr No S	**Current Cond. p1b**	**Current Cond. p5f**	**Alt. 1 p1b**	**Alt. 1 p5f**	**Alt. 2 p1b**	**Alt. 2 p5f**
1	0,7346840	0,773622099	0,62758924	0,6734615	0,890158816	0,91206985
2	0,7346930	0,736771443	0,63155554	0,6368085	0,904654951	0,90572332
3	0,7346930	0,734780443	0,63155663	0,6320988	0,904673326	0,90480911
4	0,7346930	0,734696672	0,63155665	0,6316113	0,904674665	0,90469181
5	0,7346930	0,73469319	0,63155665	0,6315621	0,904674832	0,90467699
6	0,7346930	0,734693046	0,63155665	0,6315572	0,904674853	0,90467512
7	0,7346930	0,73469304	0,63155665	0,6315567	0,904674855	0,90467489
8	0,7346930	0,73469304	0,63155665	0,6315566	0,904674856	0,90467486
9	0,7346930	0,73469304	0,63155665	0,6315566	0,904674856	0,904674856
10	0,7346930	0,73469304	0,63155665	0,6315566	0,904674856	0,90467485

4.2.2. Starvations, Blockages and WIP

Following the calculations of the production rate $(\widehat{PR})$, the numeric values of blockages $(\widehat{BL_i})$ (Formula-18), starvations $(\widehat{ST_i})$ (formula 19) and work in process inventories $(\widehat{WIP_i})$ (formula 17) are found. Results of the blockages and starvations of the machines for current and alternative materials are shown on Table 4 while the WIP data are shown on Table 5.

Table 4. Blockage and Starvation Values of Current and Alternative Cases

	Current Material		**Alternative Case 1**		**Alternative Case 2**	
	Blockage State (BL)	***Starvation State (ST)***	***Blockage State (BL))***	***Starvation State (ST)***	***Blockage State (BL)***	***Starvation State (ST)***
1	0,16530	0	0,16844	0	0,045325	0
2	0,16530	0,0000012	0,168415	0,000035	0,045283	0,000044
3	0	0,06530	0,000000475	0,118442	0,000052	0,0452756
4	0,005425	0,211086	0,01658	0,205878	0,045325	0,0383610
5	0	0,16530	0	0,168443	0	0,0453251

Table 5. WIP Values of Current and Alternative Cases

	WIP Stocks		
Buffer No	Current Material	Alternative Material 1	Alternative Material 2
1	9,55561765	9,25165291	9,01035257
2	0,9183663	0,84207608	0,95234139
3	0,92871252	1,15270514	2,25152895
4	1,11218218	1,20005987	1,53921198

5. Conclusions

Current and alternative scenarios of a production line showing Bernoulli characteristics in the packaging department of a factory are chosen in this work. To find the material that will give the optimum results in the factory environment, current and two alternative materials are tested, the output data of the test productions of the 3 materials are noted and discussed by looking at the figures comparatively.

By using two alternative materials (in addition to the current material under use), it is concluded that alternative 2 is giving superior results than the other options.

When the existing and the alternative cases are compared it is figured out that:

- As the iteration no (s) rises, it is concluded that the p_1^b and p_5^f values approach to each other and become equal after an s iteration no is reached. As it is shown at equation 14, as the $p_1^b$and p_5^f values become equal after an iteration point, it is called the "Production Rate" after on. From this point of view Table 3 shows that current material is better than alternative material 1 but alternative material 2 is better than both current and alternative material 1 (Alternative 2 PR=0,904 > Current Material PR=0,7346 > Alternative 1 PR=0,6315).
- Current and the alternative materials' choice does not significantly affect the WIP performance characteristics. Alternative material 2 tends to accumulate 1 more unit in buffer three. But it's found that the WIP ratio of material 2 is still too low than the max allowed capacity of that buffer.
- According to "blockages" performance criteria, alternative case 2 is found to be the best among the 3 cases even though it has a small surplus at machine 4.
- According to the "starvations" performance criteria, alternative case 2 is found to show better performance than the other two options by having the least starvation values.

After the analysis of the outputs, alternative 2 is concluded to be the best option. The material tested in alternative 2 is decided to be recommended for use in the packaging department instead of current material after the analysis.

The aggregate model shown in this paper including a Bernoulli line will definitely be affected from factors related to machines like part feeding times, machine cycle times, failure rates,, buffer sizes, quality of the materials as well as other factors such as environmental conditions. Currently these topics are studied under unreliability analysis and they use different assumptions and methods. As the variations of materials and environmental factors are included in the models, these models are expected to be applied more frequently. Even the methodology and calculations are complex and hard to apply, todays technological advances offer great chances to handle those problems. With the help of computers, these approaches are members to gain more popularity over time.

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